

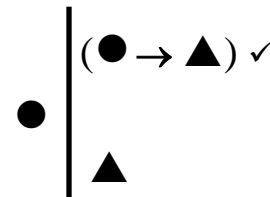
3.5. Truth Trees: Conditionals and Biconditionals

Having in the previous chapter translated truth-table-style semantic rules into truth tree format, doing so for added Chapter Three connectives poses no surprises.

1. Conditional Truth Trees. The conditional rule states that a conditional is only false in one case: where the antecedent is true, but the consequent is false. So the truth tree rule says the same, in vertical line notation.

●	▲	(● → ▲)
1	1	1
⇒ 1	0	0 ⇐
0	1	1
0	0	1

False Conditional



There are three ways to make the conditional true.

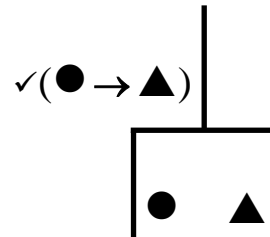
●	▲	(● → ▲)
1	1	1 ⇐
1	0	0
0	1	1 ⇐
0	0	1 ⇐

But as with true disjunctions and false conjunctions, we can distill these three cases down to two. For whenever the **conditional** is **true** either the **antecedent is false** (Valuations 3 and 4) **or** the **consequent is true** (Valuations 1 and 3) – possibly both (as in Valuation 3, which is just the overlap of these two cases).

That states the truth tree rule for true conditionals.

●	▲	$(\bullet \rightarrow \blacktriangle)$	
1	1	1	⇐
1	0	0	
0	1	1	⇐
0	0	1	⇐

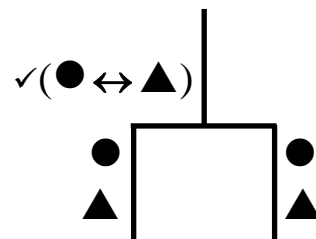
True Conditional



2. Biconditional Truth Trees. Biconditionals will prove our branchiest connective. They branch when the biconditional is true (because there are two ways to make a biconditional true).

●	▲	$(\bullet \leftrightarrow \blacktriangle)$	
1	1	1	⇐
1	0	0	
0	1	0	
0	0	1	⇐

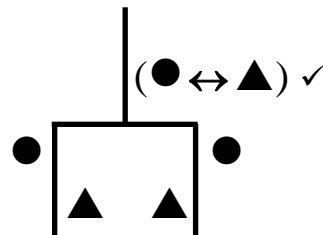
True Biconditional



And they branch when the biconditional is false (because there are two ways to make a biconditional false).

●	▲	$(\bullet \leftrightarrow \blacktriangle)$	
1	1	1	
1	0	0	⇐
0	1	0	⇐
0	0	1	

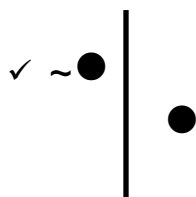
False Biconditional



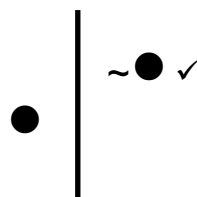
All of the procedures for testing with truth trees remain unchanged from the previous chapter. In particular, we will break down non-branching sentences before branching sentences. So **false conditionals** will go in the **non-branching** bin, while **true conditionals** and **all biconditionals** go in the **branching** bin.

Summary: Chapter Three Truth Tree Rules

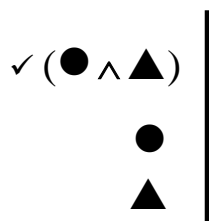
True Negation



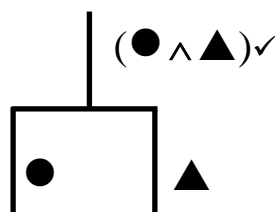
False Negation



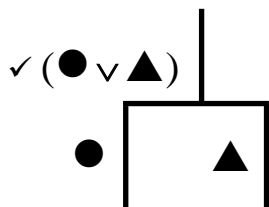
True Conjunction



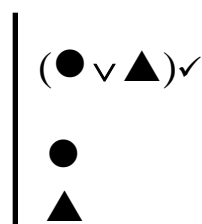
False Conjunction

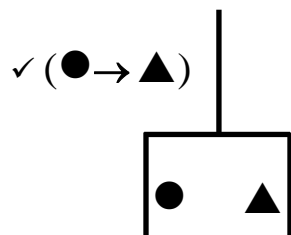
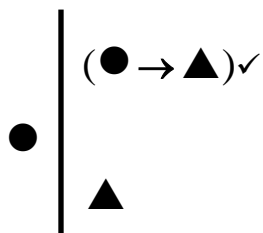
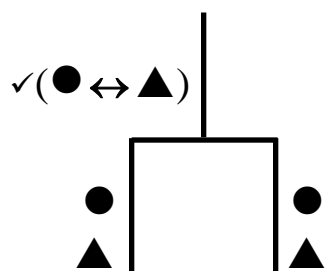


True Disjunction



False Disjunction



True Conditional**False Conditional****True Biconditional****False Biconditional**